

# Applied Mathematics for Malting and Brewing Technologists

Technological Calculations, Benchmarks and Correlations for Process Optimization

Prof. Dr. sc. techn. Gerolf Annemüller  
Dr. sc. techn. Hans-J. Manger



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Contact to the authors:

Prof. Dr. sc. techn. Gerolf Annemüller  
Buschiner Str. 34 A  
12683 Berlin  
Germany  
E-Mail: [g.annemueller@t-online.de](mailto:g.annemueller@t-online.de)

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